

# Chaos and Possible Underpinnings for Quantum Mechanics

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Deterministic chaos with its nonlinearities can formally reproduce some of the counterintuitive idiosyncrasies of quantum mechanics. In this paper I raise the question as to whether chaos might possibly supply some of the “hidden variables” that have been sought as underpinnings at a deeper level of quantum mechanics.

**Key words:** Deterministic Chaos; Foundations of Quantum Mechanics; Nonlinear Quantum Mechanics.

## 1. Preamble

Since the mid-1930's the Copenhagen interpretation of quantum mechanics has generally held sway, despite vocal objections from Einstein, Bohm, and their followers, who insisted that quantum mechanics per se be considered a provisional discipline. In the last several decades, however, partially prodded by investigations into Bell's inequality [1], there has been renewed interest in attempting to understand exactly what lies at the heart of quantum mechanics – in other words, comprehending what quantum mechanics “really means,” as opposed to using it simply as the superbly successful recipe/program that it has proven to be. The present Workshop is a prime example of such inquiry.

A summary of the quandary was put forth by Roger Penrose: “I have made no bones of the fact that I believe that the resolution of the puzzles of quantum theory must lie in our finding an improved theory. ...But even if one believes that the theory is somehow to be modified, the constraints on *how* one might do this are enormous. Perhaps some kind of “hidden variable” viewpoint will eventually turn out to be acceptable. ...” [2]. Indeed, various hidden variable theories have always been the most popular counter-interpretations [3], despite seemingly rigorous condemnation of their feasibility [4], and they have begun to re-proliferate. Nevertheless, none has met with anything but grudging, limited acceptance – and that only when applied to the simplest, abstract and hardly physical situations.

During the last decades the new science of Chaos has been applied successfully to an enormous range of scientific fields. However, its application to quantum mechanics has met with only limited success [5] – chaotic quantum systems for the most part actually appear simpler than their corresponding classical systems! Part of the problem could be that here chaos and quantum mechanics have been conceived as equals, as explanations occurring at the same level. What if chaos and its related complexity sciences were applied instead to the puzzles and paradoxes lying at the foundations of quantum mechanics? That is the question raised by this article. And it should be emphasized that *the primary intent is merely to raise the question* – at this point it is impossible to supply definitive answers or watertight examples.

## 2. Deterministic Chaos

Deterministic chaos is nonlinear science – specifically, nonlinear dynamics, involving feedback. Originating in its modern guise with the meteorological work of Lorenz in the 1960's, making it a relative newcomer, it has quickly permeated most scientific fields. Its two most broadsweeping conclusions seem to be that: 1) Nature is fundamentally nonlinear rather than linear – our compact, linear models are but a small subset of the complex, nonlinear models needed to describe nature adequately. 2) Seemingly random, indeterminate behavior can result from strictly deterministic dynamical equations. It is not truly random – that would obviate

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the usefulness of chaos – but it can be effectively unpredictable under most physical (and calculative) circumstances.

It is impossible to give much detail of such a broad science here; for our purposes we focus on three characteristics of deterministic chaos [6].

#### A) Extreme Sensitivity to Initial Conditions

(“The Butterfly Effect,” whereby, say, a butterfly’s flapping its wings in South America presumably could trigger a storm in the United States.) The boundaries between basins of attraction in strange attractors (the norm in chaos) are fractal-like (self-similar/affine), and even measurements of high precision are insufficient to guarantee a “predictable” result in the long run, since the phase-space trajectories can diverge wildly. This means that on a microscopic scale conditions such as the Uncertainty Principle can play a significant role in producing indeterminate behavior.

#### B) Periodic Points and Mixing

(“Ergodic” behavior.) For systems described by quadratic (and higher-order) equations, the behavior depends sensitively on “control parameters.” Depending on their values, one can obtain a single attractor (stable behavior settling down to a single predictable point), simple or complicated cyclic behavior, or ergodic behavior, in which all possible available values can be visited. Thus, states can be mixed in a myriad of manners.

#### C) Bifurcation and Period Doubling

(“Order in Chaos.”) Again, depending on the value(s) of the control parameters, one can reach regions in which the system goes in and out of chaotic behavior in an inordinately complex *but, in principle, predictable* fashion. For example, for a wide range of quadratic-like iterators (all iterators having a single maximum), the diminishing distance between one bifurcation and the next is given by the Grossmann-Feigenbaum constant [7],  $\delta = \delta_k / \delta_{k+1} = 4.669201609\dots$ , which many consider to be important enough to rank as a new universal constant. Figure 1 shows an example of a bifurcation diagram for the simple quadratic iterator, in which windows of order in chaos can be clearly seen. Note that this diagram is fractal, self-similar in nature – no matter how high the magnification, more and more windows of order in chaos keep appearing.

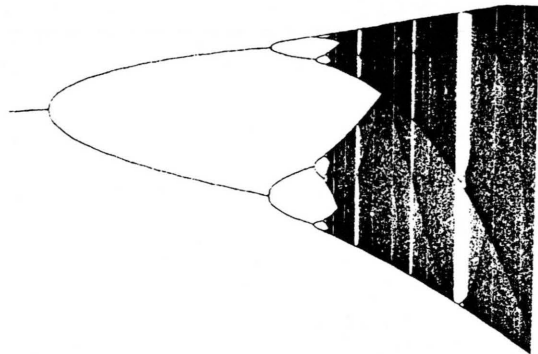


Fig. 1. Bifurcation diagram (Fig. 10 in [7]) for the simple quadratic iterator. The final values (attractors) are plotted against the control parameter,  $a$ , which ranges from about 2.5 to 4. As  $a$  increases, the single attractor point bifurcates into two, which bifurcate into four, etc., until the strange attractor, chaotic region is reached. This diagram is a fractal and is self-similar in nature: Enlarging a portion in the chaotic region will produce a copy of the whole, ad infinitum, and there will always be windows of order amid the chaos.

### 3. Some Applications Relevant to Quantum Mechanics

As of yet we have no global proof that chaotic, non-linear mechanisms lie at the heart of quantum mechanics. However, deterministic chaos can provide tidy, if qualitative, explanations of selected aspects of quantum mechanics. The following examples demonstrate a few such chaotic, alternative explanations.

#### A) Small Variations in Initial Values Lead to Exponential Decay Laws

Since a transition probability is basically a quadratic function, we can mock up its behavior with the normalized ( $0 \leq x \leq 1$ ) simple quadratic iterator,

$$x_{n+1} = ax_n(1 - x_n). \quad (1)$$

The problem is to calculate the probability of “escape” into a small interval of  $x$ ,  $\Delta J$ , (the final state) when starting from an initial small interval of  $x$ ,  $\Delta I$  (the initial state), where the width of  $\Delta I$  is determined, for example, by the Uncertainty Principle. Choosing the control parameter,  $a = 4$ , well within the chaotic region, we calculate the escape probability for points within  $\Delta I$  by graphical iteration (using the diagonal of the graph of  $x_{n+1}$  vs.  $x_n$  to bounce output from  $n$  into input of  $n + 1$ ; for details cf. [6]). For the specific

choices of  $\Delta I = [0.2, 0.2 + 10^{-11}]$  and  $\Delta J = [0.68, 0.69]$ , and with 10,000 randomly chosen initial points within  $\Delta I$ , we calculate how many iterations it takes (i.e., how long it takes) for various orbits (iterative paths followed from various initial points) to escape into  $\Delta J$ . Naturally, some orbits take longer to escape than others. We find that it takes about 40 iterations before any orbits escape; then suddenly around 62 escape on the next iteration. And when roughly half of the original points have led to escaping orbits, we find that the rate of escape has dropped to half its original value. A detailed graphical analysis leads to the exponential decay law,

$$S(n) = k e^{-n/\tau} \quad (2)$$

with an average lifetime,  $\tau \approx 140$  iterations, for this particular example. The physical correspondence can be interpreted as follows:  $\Delta I$  represents the original state, including its width consistent with the Uncertainty Principle;  $\Delta J$  represents the potential overlap with the final state; and the orbit represents the dynamics of the transition, such as number of oscillations in an atomic or nuclear “antenna” or the number of tunneling attempts of an  $\alpha$  particle. Calculations are presently underway to try to duplicate the transition rates for known well-behaved collective nuclear  $\gamma$  transitions. (Incidentally, if this explanation turns out to be valid, the initial transient period has implications for the breakdown of exponential decay at extremely short times compared with the half-life; cf. [8]).

#### B) The Existence of Attractors Implies Quantization

A single attractor implies a single “bound” state; multiple attractors in complicated patterns, as depicted, for example, in Fig. 1, lead to a whole spectrum of quantized states; and ergodic behavior could represent the continuum. Unfortunately, tying this abstract picture down to concrete examples promises to be an extraordinarily difficult task. Given a quadratic (or higher-order) iterator or nonlinear differential equation, it is a relatively straightforward numerical exercise to work out the corresponding spectrum, but the converse is not true. Thus far “spectra” have been calculated for only the simplest, highly symmetrical (and not necessarily physically meaningful) systems. A major barrier to be surmounted is just finding, let alone solving, the relevant iterators or equations for selected, known “simple” physical systems such as the hydrogen atom or an even-even liquid-drop nucleus.

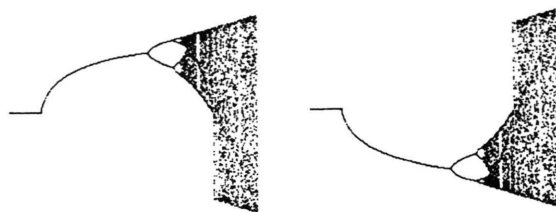


Fig. 2. Bifurcation diagram for a cubic iterator. *Left*: Starting with positive initial values of  $x$ . *Right*: Starting with negative values of  $x$ .

#### C) “Order in Chaos” Can Lead to Duality

This should be a somewhat more tractable problem. The windows of order in a bifurcation diagram (e.g., Fig. 1) predict behavior that apes duality. For example, a qualitative explanation of the two-slit experiment can be outlined as follows:

For a particle passing through a single slit, the dynamical equations are first-order, so there is no possibility of chaos. This leads to “classical,” i.e., particle-like behavior. With two slits open, however, the dynamical equations can be reduced to a single second-order equation, yielding the concurrent possibility of windows of order amid chaos, the particular behavior depending on particle position. This can mimic a diffraction-like pattern – hence, wave-like behavior and “duality” resulting from deterministic dynamics.

#### D) Spontaneous Symmetry Breaking Arises from Odd-Order Iterators

Half of the bifurcation diagram for a cubic iterator (for positive initial value of  $x$ ) is shown in Fig. 2, left; the other half (for negative initial values of  $x$ ) is a mirror image reflected through a horizontal plane. Note that, whereas in the discrete region it preserves parity, well into the chaotic region it begins to mix positive and negative values. This spontaneous breaking of symmetry is well established by classical experiments in chaotic systems, e.g., the mixing of different powders in tumblers [9]. Mixed powders spontaneously separate into bands, often of differing parities, at first glance contrary to the Second Law of Thermodynamics. This appears to occur only in chaotic domains, although the mathematics has not been worked out. It raises the possibility, however, that weak interactions such as  $\beta$  decay may not conserve parity because of chaotic odd-iterator symmetry breaking.

E) *“Action at a Distance” Can Be Simulated  
by Parallel Chaotic Orbits*

Here one enters a more speculative realm, but a “mundane” example of this can be understood in the growth of a snowflake. All snowflakes are hexagonally symmetrical; yet no two snowflakes are exactly alike. The mechanism by which a snowflake grows is freezing of the outer layer, followed by buds bursting through in positions of instability – a highly nonlinear process.

The infinite variety of snowflakes can be explained through extreme sensitivity to initial conditions, but why are the six sides all identical? How is information transmitted from one facet to another through relatively enormous distances (compared with molecular dimensions?) It cannot be simply that the six sides experience identical conditions during formation, for consider that for a falling, growing snowflake the leading edge must experience different conditions from the trailing edge – from frictional drag from the atmosphere if nothing else. A satisfying explanation is that the six sides follow identical, “parallel” orbits through “chemical phase space” [10]. This obviates the need for exchanging information or action at a distance. (Note that this is different – considerably simpler – from biological growth, which involves chemical messengers, although it is well established that biological growth involves chaos.)

A highly speculative future task would be to investigate the Bell inequality from the viewpoint of possible parallel phase-space orbits. Removing the need for action at a distance could well change one’s interpretation.

#### 4. Conclusion

The question has thus been raised. It is conceivable that nonlinear, chaotic processes can explain some of the counterintuitive Mysteries, Puzzles, and Paradoxes in quantum mechanics?! I would like to emphasize that, other than for the exponential decay problem, very little has been done in the way of calculations. Indeed, proper investigation into the points raised here would require multi-lifetimes of research effort – and multi-generations of more powerful computers. Even if the concept of chaos’ underpinnings at the foundations of quantum mechanics were to be proven valid, it is a concept unlikely to meet with day to day scientific use because of its complex, difficult mathematics. (Just as one does not perform quantum mechanical calculations when classical dynamics will suffice.) Thus, it will probably remain more important as a possible philosophical interpretation of the meaning of quantum mechanics. As such, it is a much more intuitive, satisfying explanation than the observer-dominated Copenhagen interpretation – and certainly more gratifying than quasi-science fiction ideas such as multi-universes.

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